Nuclear and Particle Physics - Problem Set 6 - Solution

Problem 1)

a) There are 6 quarks (u, d, s, c, b, t) and 6 leptons (e, μ, τ and their corresponding neutrinos ν_e, ν_μ, ν_τ), plus gluons, photons, W's, Z^0 plus the Higgs boson, for a total of 17. Actually, one can argue there are only 16 because the photon and the Z^0 are linear combinations of the W^0 and B^0 bosons, so with 3 different W states (which I didn't count separately) and the B^0, there are only 3 gauge bosons and one Higgs.
This number becomes much larger if we count different color charges (red, green, blue for quarks and any of 8 combinations of color and anti-color for the gluon) – this would yield 18 quarks, 6 leptons, 8 gluons, 1 photon, 3 weak gauge bosons (now distinguishing between the different W's) and the Higgs, for a total of 37. I will accept any number you come up with if you justify it.

b) There are 6 antiquarks and 6 antileptons, so there are 12 more (or 24 more counting anti-colors). The gluons, the photon and the Z^0 (or W^0 and B^0) are their own antiparticles, and the W^- is the antiparticle of the W^+, so they don't count extra.

c) Baryons: proton p (UUD), neutron n (UDD), lambda (UDS).
Mesons: π^+ (UD-bar), K^+ (US-bar), K^0 (DS-bar).

d) It decays into an electron, an electron-antineutrino and a muon-neutrino. Charge and energy conservation require that there is a lighter charged lepton in the final state, which can only be an electron. Then conservation of the individual lepton numbers (electron and muon number) requires the presence of both neutrinos in the final state.

e) Because the Weak Interaction is mediated by the exchange of very heavy bosons (W's and Z's which weigh about 90 GeV), its range is extremely short, much shorter than the typical distance of quarks inside the nucleus or electrons inside the atom. Therefore, the probability of an interaction is very tiny. Another way of saying this is the propagator for the W's is suppressed by a factor $Q^2/(Q^2 + M_W^2)$ relative to that of the photon.

f) Gravity and the Weak Interaction.

g) No. As the name “boson” (and their spin of 1) implies, they follow Bose-Einstein statistics, not Fermi-Pauli statistics. Therefore, as many as want to can all pile into the same quantum mechanical state.

h) For example, the c can decay into a strange quark s by emitting a W^+ boson (weak interaction), which in turn can decay into a u and d-bar pair (or a positron plus electron-neutrino).

i) Here are some of the remaining puzzles that the Standard Model doesn’t explain:
   a. Why are there 3 families of quarks and 3 families of leptons, and why do they have the charges they have?
   b. Mass hierarchy – why are there so vastly different masses (from sub-eV neutrinos to 178 GeV top quarks)?
   c. Interaction strengths: Why are there such large differences between the strength of the electroweak and the strong interactions?
   d. Where do **all** the different “fundamental constants” come from?
   e. Are neutrinos their own antiparticles (“Majorana”) or are anti-neutrinos really distinct from neutrinos (“Dirac”)?
   f. Why is there a difference in the abundance of particles vs. antiparticle in the Universe? (This may have to do with CP violation – but we also don’t understand where **that** comes from except for a tiny contribution from quark mixing).
   g. From astronomical observations, we know there is both “dark matter” and “dark energy” that we cannot explain.
   h. Gravity cannot be smoothly integrated into the standard model – indeed, we are still looking for a consistent quantum field theory including gravity.

j) The Higgs boson, which is needed to explain the generation of mass for the fundamental fermions and weak gauge bosons, has spin 0. (The graviton also has a different spin – 2 – if it exists, but it’s not part of the Standard Model).